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A FUZZY DATA ENVELOPMENT ANALYSIS METHOD FOR PERFORMANCE EVALUATION OF RENEWABLE FEEDSTOCK SUPPLIERS

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ABSTRACT
Increasing energy consumption in the world has increased the desire to find new ways to generate energy. Microalgae is new promising energy source and is regarded as a renewable feedstock for biodiesel production. Because of containing high content of non-edible oil, it has attracted the attention of many researchers in recent years. In this study, we investigate growth indicators of algae cultivation for assessing the performance of the candidate places for algae cultivation under uncertain condition. We utilize a fuzzy data envelopment analysis (FDEA) model for finding the optimum locations among the available alternatives. The model is a non-radial and non-oriented one and evaluates each candidate under uncertainty. An equivalent crisp linear programming problem is formulated for solving the suggested FDEA model under various levels of uncertainty. Then, an actual case study is provided in Iran in order to validate the proposed approach.

Keywords: Renewable feedstock; Fuzzy data envelopment analysis; Efficiency measurement, Uncertainty; renewable feedstock suppliers

1. INTRODUCTION

Human energy needs are increasing rapidly due to the rapid pace of technological advancements. All countries need energy for a variety of purposes. It has been proved that fuels derived from biomass can be used in the transportation system instead of conventional fossil fuel. The United States can use its biofuel production capacity to meet 30 percent of its fossil fuel needs (Yue et al., 2014). Various countries use different alternative energy sources, including solar power, hydro energy, wind energy, tidal energy, geothermal energy, and fuel procured from biomass resources. Biomass has the potential to provide more than a quarter of the world's energy needs (Rawat et al., 2011). More than four-fifths of the energy needed by humans comes from fossil fuel sources. These resources include natural gas, oil, and coal. This demand for energy causes the economies of countries to be affected by the availability and price of this resource. It is noteworthy that the growth rate of energy demand in the last decade is higher than the growth rate of the population. (Ong et al., 2011). A variety of biomass can be used to produce biofuel. On the other hand, FAO does not recommend the use of edible biomass sources as raw material for biofuel production. Hence, we have the challenge of replacing non-edible feedstock with edible feedstock to prevent food crisis growth. Recently, Jatropha and microalgae plants are utilized as a feedstock for various biofuel production, including biodiesel. The mentioned feedstocks are non-edible containing high oil materials, which leads to more biodiesel production (Kumar and Sharma, 2011).

Qin et al. (2012) highlighted that the economies depend on fossil fuel resources to follow some political, economic, and environmental issues. The population of people using biomass in traditional ways will increase to 100 million by the year 2030 (for more details see United Nations Development Program (UNDP), 2002). There are a variety of sources for biofuel production, such as forest residues, agricultural residues, and non-edible energy crops. (see International Energy Agency (IEA), 2014 and Intergovernmental Panel on Climate Change (IPCC), 2011). Table 1 shows the approximate global capacity of bioenergy production.

Table 1. Bioenergy generation from different biomass

Resource	Technical potential
Biomass	In 2050 (EJ/yr)
Production of energy crops on surplus agricultural	0-700
Production of energy crops in marginal land	<60-110
Agricultural residues	15-150
Forest residues	30-150
muck	5-55
Organic wastes	5-50
Total	<60->1100

According to IPCC (2011), in developed countries, only 5 percent of total energy demand comes from biomass resources. According to World Energy Council (WEC) (2011) report, people living in rural areas, also known as the villagers, make up more than 50 percent of the population, utilize biomass resources, generally to produce light and heat for cooking.

The light reaction process is applied to microalgae oil and bio-hydrogen to produce biodiesel (Banerjee et al., 2002). One way to deal with decreasing fossil fuel resources, as well as environmental issues, is to use renewable fuels made from oleaginous microalgae (Chisti, 2007; Hu, 2014). The high price of fossil fuels, as well as the phenomenon of global warming, are other reasons for using renewable energy sources (Nagle and Lemke, 1995). Various types of oleaginous algae can be converted into biofuels, including biodiesel. However, only specific microalgae species have been exploited in large-scale. These species include Scenedesmus, Chlorella, and Nannochloropsis (Hu et al., 2008).

There are two main reasons for paying attention to microalgae; the conversion of microalgae to biodiesel is highly efficient, and it has the potential to not only reduce air pollution but also to maintain high water quality (Rios et al., 2013). Feedstocks such as corn and sunflower can be converted into biodiesel. However, the production of biodiesel from these agricultural products incurs exorbitant costs (Leite et al., 2013). Providing energy safety leads to strengthening economies against energy price shocks (Brown and Huntington, 2008). CO_2 emissions have extremely increased since the industrial revolution (Global Greenhouse Gas Reference Network. - Swapnesh and Srivastava), which leads to global warming (IPCC 1990).

Microalgae cannot be consumed as food; hence it has a noncompetitive nature to food markets. It can also be cultivated using nutrient waste streams. Due to these features, the biodiesel produced from microalgae is called as a third-generation biofuel (Roberts et al., 2013). The energy contained by microalgae oil is almost equivalent to 80 percent of the energy contained by petroleum, which is equal to 35,800 kJ energy (Chisti, 2013). Recent researches prove that the annual harvest of algae from each is between 15 and 25 tons per hectare (Lam and Lee, 2012). The amount of lipid extracted from microalgae is about 4.5-7.5 metric tons per hectare in one year without considering growth conditions optimization (Lam and Lee, 2012). This amount is much more than the annual amount of lipid extracted from other resources including soybeans which is equal to 0.4 metric ton per hectare, palm oil which is equal to 3.62 metric ton per hectare, and Jatropha which is equal to 4.14 metric ton per hectare (Lam and Lee, 2011).

Phosphorus and nitrogen are among the essential substances needed in algal cultivation. Mentioned nutrients are typically acquired from non-organic and organic manure, leading to greenhouse gas (GHG) emissions (Lam, 2012). Also, algae cultivation requires a considerable amount of energy, water, and carbon dioxide (Dalrymple, 2013). Algae growth requires light, mineral salts, carbon dioxide, and water. Microalgae cultivation is highly efficient at a temperature between 15 to 30 °C (Molina et al., 1999).

Identifying the optimal location for algae cultivation relies on several criteria including environmental conditions of the cultivation area, economic policies, having sufficient information about the geographical conditions of the region, and knowing algae growth conditions considering its breed.

Optimal site selection for algae cultivation depends on three crucial principles: considering physical and geographical conditions for algae cultivation, considering the political situation in the region, considering the price of land (Maxwell et al., 1985). The yield of algae cultivation depends on climatic conditions such as irradiance, the amount of available CO_2, nutrients accessibility, and temperature (Darzins et al. 2010).

Studies show that the articles have not considered growth parameters and uncertainty conditions to determine optimal locations where algae growth is ideal except the article by Babazadeh et al. (2017). Nevertheless, the approach of his study does not meet sufficient capabilities in unstable conditions. To complete this shortage, the current paper objective is to identify the best places for algae cultivation considering uncertain conditions by developing a productive fuzzy data envelopment analysis (FDEA) model. To implement this method, we define the important factors for the cultivation of algae to identify appropriate locations.

The paper continues as follows: Section 2 reviews the literature of the previous efforts for performance evaluation of the Algae culture places under uncertainty. The employed methodology, along with the FDEA model, is described in Section 3. The next section provides a solution method through the possibilistic programming approach. The penultimate section describes the major proposals and sustainable development indicators of the case study and presents the obtained results by implementing our new approach. The results of this study are presented in section 6, including the managerial concepts of the achieved results.

2. LITERATURE REVIEW

One of the advantages of algae cultivation in wastewater is that it removes nutrients from water and air. Kligerman and Bouwer (2015) noted that microalgae production is economical through nitrogen and phosphorus excretion. Demirbas (2011) claims that microalgae are the only feedstock that can be converted into biodiesel.

Quinn et al. (2013) applied an engineering process model to assess the conversion cycle of algae into biofuels. The outcomes demonstrate nutrient recycling in the microalgae-to-biofuel lifecycle plays an essential role in delivering a desirable net energy ratio (NER) and GHG. It should be noted that NER refers to the energy consumed over the produced energy in the microalgae biofuel production process.

Weyer et al. (2009) determine an absolute upper limit to algal production using a theoretical method considering physical requirements and assumptions of distinct efficiencies. The results show that the maximum amount of oil that can be obtained annually from each hectare of algae is about 354,000 liters. Babazadeh et al. (2016) proposed an FDEA approach (fuzzy data envelopment analysis) to specify the optimum sites for JCL cultivation under uncertainty conditions. Bray et al. (2014) proposed a classical fuzzy DEA model to determine the impacts of uncertainly on the productivity of the considered transportation services. Zografidou et al. (2015) used a multiperiod goal programming method to optimize renewable energy production in Greece. The model was developed regarding social, ecological, and economic criteria. Toloo et al. (2018) proposed an approach that integrates both pessimistic and optimistic models into one model. The developed model is capable of determining a specific condition of each imprecise dual-role factor as well as to develop a structure for estimating an optimal reallocation model of each dual-role factor among the DMUs. Azadeh et al. (2014) applied an integrated fuzzy-DEA approach to determine optimum locations for wind plant installations. They applied principal component analysis (PCA) and numerical taxonomy (NT) approach to verify the outcomes of the developed DEA model. Emrouznejad et al. (2016) applied a multiplicative DEA model to rank several prediction methods. Bray et al. (2014) presented a feature selection analysis based on the fuzzy DEA to assess the impact of uncertainty on the efficiency of the transportation system. Wanke et al. (2015) developed an integrated FDEA and bootstrapping method to evaluate productive efficiency of banks under uncertainty. The results show that fuzziness is predominant over randomness in identifying missing variables and interpreting the results. Azadi et al. (2015) proved that by combining the two methods of DEA and Enhanced Russel Measure approach, a new model could be presented to evaluate the supply chain suppliers in terms of efficiency. Egilmez et al. (2016) proposed a new FDEA approach that could also include uncertainty conditions in the model. They applied this new FDEA approach to evaluate and rank food factories in the USA. Azadeh et al. (2016) applied an integrated approach, including FDEA, with an analytic hierarchy process (AHP) to solve a model related to a layout design problem with uncertain parameters. Wen et al. (2017) proposed three types of uncertain DEA models along with their equivalent crisp models that are applicable in uncertain environments. Shabanpour et al. (2017) developed a two-step approach based on goal programming (GP) and robust double frontiers DEA to evaluate sustainable suppliers where robust optimization model is employed to carry out the data uncertainty. Pitchipoo et al. (2018) presented a hybrid DEA approach to select suppliers in process industries. They investigated chemical processing as a case study.

3. METHODOLOGY

DEA models are mostly used for evaluating the relative efficiency of different organizations or production centers named decision-making units (DMUs). Data from various processes are associated with uncertainty due to measurement error, environmental and internal factors. Uncertainty theory plays an important role in DEA models. Gou (2009) applied a fuzzy DEA model for specifying the places of restaurants under imprecise conditions. Toloo and Nalchigar (2011) and Toloo (2014) studied the supplier's performance evaluation by the DEA method in the presence of uncertain data. Azadeh et al., (2011b) suggested utilizing DEA models under uncertainty to create reliable results. Azadeh et al. (2014) developed a hierarchical FDEA model to rank the best locations for power generation systems. Zografidou et al. (2016) applied a binary GP model to optimize the Greek renewable energy supply chain considering economic, social, and environmental aspects. The best network structure is selected through applying the DEA model. Salahi et al. (2018) developed equivalent models for the robust non-radial Russell measure and proposed its improved models under uncertainty. The uncertainty sets were interval and ellipsoidal.

The main purpose of this study is assessing the relative efficiency of different provinces of Iran for microalgae cultivation considering economic, environmental and social indicators.

The steps performed to optimize algae cultivation areas are as below:

Step 1: Identify the sustainability indicators to evaluate the performance of cultivation areas.

Step 2: Divide the sustainability indicators into good and bad outputs by using the method of Azadeh et al. (2011a). By this manner, the data is not transformed to be compatible with the aim of maximizing/minimizing the DEA approach.

Step 3. Use triangular possibility distribution for modeling the fuzzy parameters of the DEA model,

Step 4. Specify the minimum acceptable feasibility degree of the constraints and then transform the suggested FDEA model to a crisp version for ranking the whole DMUs.

Step 5. Review and validation of the results of FDEA according to the results achieved by the crisp DEA model Using Spearman's rank correlation method.

Step 6. If validation of the FDEA model is confirmed, employ the FDEA model to calculate the relative efficiency of DMUs at the desired level of α-cuts of DM. If not, the utilized FDEA model is not appropriate for calculating the performance of DMUs.

3.1 The None-Radial DEA Model

DEA is a non-parametric approach based on mathematical programming technique for accessing the technical efficiency of a set of similar DMUs (Cook et al., 2014). In the non-parametric approach there is no need to weigh the inputs and outputs, nor to set production function which are usual in statistical regression methods. However, there are some studies aimed at using parametric methods in estimating production function (Lovell and Schmidt, 1988). The parametric and non-parametric methods could be merged in a hybrid approach with better performance (Tofallis, 2001) where the DEA firstly identifies the efficient DMUs and then a suitable frontier matches it.

There are radial and non-radial approaches in order to project an inefficient DMU on the efficient frontier. The aim of a radial model is decreasing the input (increasing the output) values, as much as possible, meaning that it identifies a set of points with an identical ratio of inputs (outputs). However, a non-radial model neglects the radial characteristic of inputs and outputs. Färe et al. (2005) described in detail the advantage of dividing indicators into desirable and undesirable ones. We extend the non-radial DEA model of Sueyoshi and Goto (2011) which divides the outputs into good (desirable) and bad (undesirable) outputs. The utilized DEA model integrates the good and bad outputs in a unified model.

The nomenclatures used in the FDEA model could be represented as follows. Above the uncertain parameters is a tilde (~) mark.

Sets

j,k	Locations of cultivating algae (DMUs) $j,k = 1,\dots,n$
r	Good (desirable) outputs $r = 1,\dots,s$
f	Bad (undesirable) outputs $f = 1,\dots,h$

Parameters

$\tilde{R}_r^g$	Span of good output r
$\tilde{R}_f^b$	Span of bad output f
$\tilde{g}_{rj}$	Amount of good output r for DMU_j
$\tilde{b}_{fj}$	Amount of bad output f for DMU_j

Variables

φ_j^g	Structural variable of good outputs for DMU_j
φ_j^b	Structural variable of bad outputs for DMU_j
s_r^g	Surplus variable for good output r
s_f^b	Slack variable for bad output f
β	Relative efficiency score for DMU

We propose the following non-radial FDEA model to assess the relative efficiency of DMU_k, the unit under evaluation:

$$\max z = \sum_{r=1}^{s} \tilde{R}_r^g s_r^g + \sum_{f=1}^{h} \tilde{R}_f^b s_f^b \qquad (1)$$

s.t.

$$\sum_{j=1}^{n} \tilde{g}_{rj}\varphi_j^g - s_r^g = \tilde{g}_{rk}, \forall r = 1,\dots,s \qquad (2)$$

$$\sum_{j=1}^{n} \varphi_j^g = 1, \qquad (3)$$

$$\sum_{j=1}^{n} \tilde{b}_{fj}\varphi_j^b + s_f^b = \tilde{b}_{fk}, \forall f = 1,\dots,h \qquad (4)$$

$$\sum_{j=1}^{n} \varphi_j^b = 1, \qquad (5)$$

$$\varphi_j^g \ge 0, \varphi_j^b \ge 0, \varphi_r^g \ge 0, \varphi_f^b \ge 0. \qquad (6)$$

The model is solved *n* times to calculate the relative efficiency scores of the whole DMUs.

In the FDEA model, the structural variables are used to connect the good and bad outputs by a convex combination. The first term of Equation (2) indicates a convex combination of all desirable output vectors $\begin{pmatrix}\tilde{g}_{11}\\ \vdots \\ \tilde{g}_{s1}\end{pmatrix}, \begin{pmatrix}\tilde{g}_{12}\\ \vdots \\ \tilde{g}_{s2}\end{pmatrix}, \dots, \begin{pmatrix}\tilde{g}_{1n}\\ \vdots \\ \tilde{g}_{sn}\end{pmatrix}$ which is greater than or equal to the desirable output vector of DMU under evaluation $\begin{pmatrix}\tilde{g}_{1k}\\ \vdots \\ \tilde{g}_{sk}\end{pmatrix}$ by taking into account the non-negative surplus variable vector $\begin{pmatrix}s_1^g\\ \vdots \\ s_s^g\end{pmatrix}$. Whereas, the first term of Equation (4) designates a convex combination of all undesirable output vectors $\begin{pmatrix}\tilde{b}_{11}\\ \vdots \\ \tilde{b}_{h1}\end{pmatrix}, \begin{pmatrix}\tilde{b}_{12}\\ \vdots \\ \tilde{b}_{h2}\end{pmatrix}, \dots, \begin{pmatrix}\tilde{b}_{1n}\\ \vdots \\ \tilde{b}_{hn}\end{pmatrix}$ which is at most $\begin{pmatrix}\tilde{b}_{1k}\\ \vdots \\ b_{hk}\end{pmatrix}$ by considering the non-negative slack variable vector $\begin{pmatrix}s_1^b\\ \vdots \\ s_h^b\end{pmatrix}$. Note that the convex-combination conditions are hold by Equations (3) and (5) along with the non-negativity conditions in Equation (6). We define the fuzzy coefficients of the objective function (1) as below:

$$\tilde{R}_r^g = 1/[(1+s+h)[\max\{\tilde{g}_{rj} | j = 1,\dots,n\} - \min\{\tilde{g}_{rj} | j = 1,\dots,n\}]] \qquad (7)$$

$$\tilde{R}_f^b = 1/[(1+s+h)[\max\{\tilde{b}_{fj} | j = 1,\dots,n\} - \min\{\tilde{b}_{fj} | j = 1,\dots,n\}]] \qquad (8)$$

We will later expand Equations (7) and (8) to their equivalent crisp forms (see Equations (39) and (40)). Since there is no input in the model, we consider a dummy input for all DMUs. In Equations (7) and (8), 1 stands for the dummy input.

The relative efficiency score of DMU_k is measured by equation (9).

$$\beta = 1 - \left(\sum_{r=1}^{s} \tilde{R}_r^g s_r^{g*} + \sum_{f=1}^{s} \tilde{R}_f^b s_f^{b*}\right) \tag{9}$$

The star sign "*" shows the optimality condition.

Since the good and bad outputs of the FDEA model are uncertain, it is not possible to solve this model by linear programming (LP) techniques. To tackle this issue, we transfer the model to an equivalent crisp model. Triangular possibility distribution is used to model the distribution of fuzzy good and bad outputs. To make the triangular possibility distribution, the limited historical data and knowledge of experts are utilized. The next section employs some recent methods in possibilistic programming approach to develop an appropriate methodology for dealing with ambiguousness in the FDEA model.

4. THE SOLVING PROCEDURE

This section transfers the suggested FDEA model to an equivalent crisp model. Toward this end, we utilize the possibilistic programming method to deal with the fuzzy coefficients in objective function and constraints. Some methods such as flexible programming (Dubois et al., 2003) and compromise programming (Parra et al., 2005) are used to convert a possibilistic programming model into its equivalent crisp model. Nevertheless, the method proposed by Jiménez et al. (2007) is employed in this paper. The advantages of this method include: (i) The method is efficient from computational complexity point of view; (ii) Utilizing the ranking method developed by Jiménez (1996), it could be used to construct different linear and non-linear membership functions; and (iii) the method uses expected interval (EI) along with expected value (EV) of fuzzy numbers to create the equivalent crisp model. For more details, see Pishvaee and Torabi (2010).

Now, we describe the required principles to transform the FDEA model to its equivalent crisp model. Assume that $\tilde{c}$ is a triangular fuzzy number which its possibility distribution is determined by three points i.e.,$\tilde{c} = (c^p, c^m, c^o)$, where $c^m, c^p,$ and c^o represent the most possible, most pessimistic, and most optimistic values, respectively. The membership function of $\tilde{c}$ is denoted by $\mu_{\tilde{c}}$ which is a continuous function from $\mathbb{R}$ to [0,1] as below:

$$\mu_{\tilde{c}} = \begin{cases} 0 & \text{if } x \in (-\infty, c^p] \\ f_c(x) = \dfrac{x - c^p}{c^m - c^p} & \text{if } x \in [c^p, c^m] \\ 1 & \text{if } x = c^m \\ g_c(x) = \dfrac{c^o - x}{c^o - c^m} & \text{if } x \in [c^m, c^o] \\ 0 & \text{if } x \in [c^p, +\infty) \end{cases} \tag{10}$$

The α-cut set of fuzzy number $\tilde{c}$ can be defined as $c_\alpha = \{x \in \Omega | \mu_{\tilde{c}}(x) \geq \alpha\}$ where Ω is the universe set. Since $\mu_{\tilde{c}}$ is continuous, the α-cut sets are closed and bounded and can be presented as $c_\alpha = [f_c^{-1}(\alpha), g_c^{-1}(\alpha)]$. According to, The EI and EV of fuzzy number $\tilde{c}$ can be written as follows, respectively: (see Heilpern, 1992, and Jimenez, 1996):

$$\begin{aligned} EI(\tilde{c})[E_1^c, E_2^c] \\ &= \left[\int_0^1 f_c^{-1}(x)dx, \int_0^1 g_c^{-1}(x)dx\right] \\ &= [\tfrac{1}{2}(c^p + c^m), \tfrac{1}{2}(c^m + c^o)] \end{aligned} \tag{11}$$

$$EV(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^o}{4} \tag{12}$$

Note that the EV is the midpoint of the EI.

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers. Dubois and Prade (1978) applied the following interval in order to use the Zadeh's (1978) minimum extension principle for aggregating $\tilde{a}$ and $\tilde{b}$:

$$\begin{aligned} [f_{\lambda a + \gamma b}^{-1}(x), g_{\lambda a + \gamma b}^{-1}(x)] \\ &= [\lambda f_a^{-1}(x) \\ &\quad + \gamma f_b^{-1}(x), \lambda g_a^{-1}(x) \\ &\quad + \gamma g_b^{-1}(x)] \end{aligned} \tag{13}$$

Accordingly, Jiménez et al. (2007) suggested the followings EI and EV:

$$EI(\lambda\tilde{a} + \gamma\tilde{b}) = \lambda EI(\tilde{a}) + \gamma EI(\tilde{b}) \tag{14}$$

$$EV(\lambda\tilde{a} + \gamma\tilde{b}) = \lambda EV(\tilde{a}) + \gamma EV(\tilde{b}) \tag{15}$$

Furthermore, the following function verifies how much $\tilde{a}$ is bigger than $\tilde{b}$ (see Jiménez (1996)):

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \tag{16}$$

where $[E_2^a - E_1^a]$ and $[E_2^b - E_1^b]$ are the EIs of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, respectively. If $\mu_M(\tilde{a}, \tilde{b}) \geq \alpha$, then $\tilde{a}$ is greater than or equal to $\tilde{b}$, at least at level α which is illustrated by $\tilde{a} \geq_\alpha \tilde{b}$.

Moreover, $\tilde{a}$ is indifferent (equal) to $\tilde{b}$ in degree of α if the inequality (17) is met at the same time (Parra et al., 2005):

$$\tilde{a} \approx_\alpha \tilde{b} \tag{17}$$

Inequality (17) can be rewritten as follows:

$$\frac{\alpha}{2} \leq \mu_M(\tilde{a}, \tilde{b}) \leq 1 - \frac{\alpha}{2} \tag{18}$$

Now, we employ the aforementioned concepts to develop an auxiliary crisp model for the following general possibilistic programming model:

$$\begin{aligned} &\min z = \tilde{\boldsymbol{c}}\boldsymbol{x} \\ &\text{s.t.} \\ &\tilde{\boldsymbol{a}}_i \boldsymbol{x} \geq \tilde{b}_i \qquad i = 1, \dots, l \\ &\tilde{\boldsymbol{a}}_i \boldsymbol{x} = \tilde{b}_i \qquad i = l+1, \dots, m \\ &\boldsymbol{x} \geq \boldsymbol{0}_n \end{aligned} \tag{19}$$

where $\tilde{\boldsymbol{c}} = (\tilde{c}_1, \dots, \tilde{c}_n)$ is the fuzzy cost coefficients vector, $\tilde{\boldsymbol{a}}_i = (\tilde{a}_{i1}, \dots, \tilde{a}_{in})$ is i^{th} row vector of the constraint matrix for $i = 1, \dots, m$, $\boldsymbol{x} = (x_1, \dots, x_n)$ is decision variables vector, and $\boldsymbol{0}_n$ is the origin in the $n-$dimensional (real) Euclidean space, i.e, $\boldsymbol{0}_n = (0, \dots, 0) \in \mathbb{R}^n$.

A decision variables vector $\boldsymbol{x} \in \mathbb{R}^n$ is feasible in degree of α (Jiménez et al., 2007) if $\min\{\mu_M(\tilde{\boldsymbol{a}}_i\boldsymbol{x}, \tilde{b}_i) | i = 1, \dots, m\} = \alpha$. In other words, $\boldsymbol{x}$ is α-feasible when:

$$\tilde{\boldsymbol{a}}_i\boldsymbol{x} \geq_\alpha \tilde{b}_i, \quad i = 1, \dots, l \tag{20}$$

which leads to (see formula 16):

$$\frac{E_2^{\boldsymbol{a}_i x} - E_1^{b_i}}{E_2^{\boldsymbol{a}_i x} - E_1^{b_i} - \left(E_1^{\boldsymbol{a}_i x} - E_2^{b_i}\right)} \geq \alpha, \quad i = 1, \dots, l \tag{21}$$

or equivalently (see 15):

$$\left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, \quad i = 1, \dots, l \tag{22}$$

Now, we extend the above procedure for fuzzy equalities. When two fuzzy numbers $\tilde{\boldsymbol{a}}_i\boldsymbol{x}$ and $\tilde{b}_i$ are α $-$equal:

$$\tilde{\boldsymbol{a}}_i\boldsymbol{x} \approx_\alpha \tilde{b}_i, \qquad i = l+1, \dots, m \tag{23}$$

we have

$$\frac{\alpha}{2} \leq \frac{E_2^{\boldsymbol{a}_i\boldsymbol{x}} - E_1^{b_i}}{E_2^{\boldsymbol{a}_i\boldsymbol{x}} - E_1^{b_i} - \left(E_1^{\boldsymbol{a}_i\boldsymbol{x}} - E_2^{b_i}\right)} \leq 1 - \frac{\alpha}{2}, \quad i = l+1, \dots, m \tag{24}$$

From (15), the above inequalities are rewritten as follows:

$$\left[\left(1-\frac{\alpha}{2}\right)E_2^{\boldsymbol{a}_i} + \frac{\alpha}{2}E_1^{\boldsymbol{a}_i}\right]\boldsymbol{x} \geq \frac{\alpha}{2}E_2^{b_i} + \left(1-\frac{\alpha}{2}\right)\frac{\alpha}{2}E_1^{b_i}, \quad i = l+1, \dots, m \tag{25}$$

$$\left[\frac{\alpha}{2}E_2^{a_i} + \left(1-\frac{\alpha}{2}\right)E_1^{a_i}\right]\boldsymbol{x} \leq \left(1-\frac{\alpha}{2}\right)E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, \quad i = l+1, \dots, m \tag{26}$$

The following definition is used to convert the fuzzy objective function of model (19) into its equivalent crisp model (Jiménez et al., 2007). A feasible vector like $\boldsymbol{x}^0$ is an α-acceptable optimal solution for model (19) if and only if for all feasible decisions vectors $\boldsymbol{x}$ we have

$$\tilde{\boldsymbol{c}}\boldsymbol{x} \geq_{\frac{1}{2}} \tilde{\boldsymbol{c}}\boldsymbol{x}^0 \tag{27}$$

or (see 22)

$$\frac{E_2^{\boldsymbol{c}x} - E_1^{\boldsymbol{c}x}}{2} \geq \frac{E_2^{\boldsymbol{c}x^0} - E_1^{\boldsymbol{c}x^0}}{2} \tag{28}$$

The feasible solution $\boldsymbol{x}$ will be hold in constraints (29)-(31):

$$\tilde{\boldsymbol{a}}_i\boldsymbol{x} \geq_\alpha \tilde{b}_i, \qquad i = 1, \dots, l \tag{29}$$
$$\tilde{\boldsymbol{a}}_i\boldsymbol{x} \approx_\alpha \tilde{b}_i, \qquad i = l+1, \dots, m \tag{30}$$
$$\boldsymbol{x} \geq \boldsymbol{0}_n \tag{31}$$

If the vector $\boldsymbol{x}^0$ is an optimal solution for the problem (32), it is an optimal α-acceptable solution for the model (19).

$$\begin{aligned}
&\min z = EV(\tilde{\boldsymbol{c}})\boldsymbol{x} \\
&\text{s.t.} \\
&\left[\left(1-\frac{\alpha}{2}\right)E_2^{\boldsymbol{a}_i} + \alpha E_1^{\boldsymbol{a}_i}\right]\boldsymbol{x} \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, \quad i = 1, \dots, l \\
&\left[\left(1-\frac{\alpha}{2}\right)E_2^{\boldsymbol{a}_i} + \frac{\alpha}{2}E_1^{\boldsymbol{a}_i}\right]\boldsymbol{x} \geq \frac{\alpha}{2}E_2^{b_i} + \left(1-\frac{\alpha}{2}\right)\frac{\alpha}{2}E_1^{b_i}, \quad i = l+1, \dots, m \\
&\left[\frac{\alpha}{2}E_2^{\boldsymbol{a}_i} + \left(1-\frac{\alpha}{2}\right)E_1^{\boldsymbol{a}_i}\right]\boldsymbol{x} \leq \left(1-\frac{\alpha}{2}\right)E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, \quad i = l+1, \dots, m \\
&\boldsymbol{x} \geq \boldsymbol{0}_n
\end{aligned} \tag{32}$$

Equations (33) and (34) calculate the maximum and minimum of n fuzzy numbers $\tilde{a}_1, \dots, \tilde{a}_n$ as follows:

$$\max(\tilde{a}_1, \dots, \tilde{a}_n)(z) = \sup_{z=\max(x_1,\dots,x_n)} \min(\tilde{a}_1(x_1), \dots, \tilde{a}_n(x_n)), \quad \forall z \in R \tag{33}$$

$$\min(\tilde{a}_1, \dots, \tilde{a}_n)(z) = \sup_{z=\min(x_1,\dots,x_n)} \min(\tilde{a}_1(x_1), \dots, \tilde{a}_n(x_n)), \quad \in R \tag{34}$$

For more details about equations (33) and (34), we refer the interested readers to Hong and Kim (2006).

According to (33) and (34), it is deduced that:

$$\max\{\tilde{g}_{rj}\} = \tilde{\boldsymbol{g}}_{r,max} = (g_{r,max}^p, g_{r,max}^m, g_{r,max}^o) \tag{35}$$
$$\min\{\tilde{g}_{rj}\} = \tilde{\boldsymbol{g}}_{r,min} = (g_{r,min}^p, g_{r,min}^m, g_{r,min}^o) \tag{36}$$
$$\max\{\tilde{b}_{fj}\} = \tilde{\boldsymbol{b}}_{f,max} = (b_{f,max}^p, b_{f,max}^m, b_{f,max}^o) \tag{37}$$
$$\min\{\tilde{b}_{fj}\} = \tilde{\boldsymbol{b}}_{f,min} = (b_{f,min}^p, b_{f,min}^m, b_{f,min}^o) \tag{38}$$

Consequently, we define $\tilde{\boldsymbol{R}}_r^g$ and $\tilde{\boldsymbol{R}}_f^b$ as bellow:

$$\tilde{\boldsymbol{R}}_r^g = \left(R_r^{gp}, R_r^{gm}, R_r^{go}\right) = \frac{1}{(1+s+h)}\left(\frac{1}{g_{r,max}^o - g_{r,min}^o}, \frac{1}{g_{r,max}^m - g_{r,min}^m}, \frac{1}{g_{r,max}^p - g_{r,min}^p}\right) \tag{39}$$

$$\tilde{\boldsymbol{R}}_f^b = \left(R_f^{bp}, R_f^{bm}, R_f^{bo}\right) = \frac{1}{(1+s+h)}\left(\frac{1}{b_{f,max}^o - b_{f,min}^o}, \frac{1}{b_{f,max}^m - b_{f,min}^m}, \frac{1}{b_{f,max}^p - b_{f,min}^p}\right) \tag{40}$$

All in all, we formulate the following crisp model which is equivalent to the FDEA model (1-6):

$$\max z = \sum_{r=1}^{s}\left(\frac{R_r^{gp}+2R_r^{gm}+R_r^{go}}{4}\right)d_r^g + \sum_{f=1}^{h}\left(\frac{R_f^{bp}+2R_f^{bm}+R_f^{bo}}{4}\right)d_f^b \tag{41}$$

s.t.

$$\sum_{j=1}^{n}\left[\left(1-\frac{\alpha}{2}\right)\left(\frac{g_{rj}^m+g_{rj}^o}{2}\right)+\left(\frac{\alpha}{2}\right)\left(\frac{g_{rj}^p+g_{rj}^m}{2}\right)\right]\lambda_j^g - d_r^g \geq \left(\frac{\alpha}{2}\right)\left(\frac{g_{rk}^m+g_{rk}^o}{2}\right)+\left(1-\frac{\alpha}{2}\right)\left(\frac{g_{rk}^p+g_{rk}^m}{2}\right), \quad \forall r \tag{42}$$

$$\sum_{j=1}^{n}\left[\left(\frac{\alpha}{2}\right)\left(\frac{g_{rj}^m+g_{rj}^o}{2}\right)+\left(1-\frac{\alpha}{2}\right)\left(\frac{g_{rj}^p+g_{rj}^m}{2}\right)\right]\lambda_j^g - d_r^g \leq \left(1-\frac{\alpha}{2}\right)\left(\frac{g_{rk}^m+g_{rk}^o}{2}\right)+\left(\frac{\alpha}{2}\right)\left(\frac{g_{rk}^p+g_{rk}^m}{2}\right), \quad \forall r \tag{43}$$

$$\sum_{j=1}^{n}\lambda_j^g = 1, \tag{44}$$

$$\sum_{j=1}^{n}\left[\left(1-\frac{\alpha}{2}\right)\left(\frac{b_{fj}^m+b_{fj}^o}{2}\right)+\left(\frac{\alpha}{2}\right)\left(\frac{b_{fj}^p+b_{fj}^m}{2}\right)\right]\lambda_j^b - d_f^b \geq \left(\frac{\alpha}{2}\right)\left(\frac{b_{fk}^m+b_{fk}^o}{2}\right)+\left(1-\frac{\alpha}{2}\right)\left(\frac{b_{fk}^p+b_{fk}^m}{2}\right), \quad \forall f \tag{45}$$

$$\sum_{j=1}^{n}\left[\left(\frac{\alpha}{2}\right)\left(\frac{b_{fj}^m+b_{fj}^o}{2}\right)+\left(1-\frac{\alpha}{2}\right)\left(\frac{b_{fj}^p+b_{fj}^m}{2}\right)\right]\lambda_j^b - d_f^b \leq \left(1-\frac{\alpha}{2}\right)\left(\frac{b_{fk}^m+b_{fk}^o}{2}\right)+\left(\frac{\alpha}{2}\right)\left(\frac{b_{fk}^p+b_{fk}^m}{2}\right), \quad \forall f \tag{46}$$

$$\sum_{j=1}^{n}\lambda_j^b = 1, \tag{47}$$

$$\lambda_j^g \geq 0, \lambda_j^b \geq 0, \lambda_r^g \geq 0, \lambda_f^b \geq 0. \tag{48}$$

Here, there are $2(s+h+1)$ constraints with $2n+s+h$ non-negative decision variables. The above equivalent crisp form of the FDEA model integrates good and bad indicators in a unified model and could efficiently deal with the fuzziness of parameters. The relative efficiency score for each DMU is achieved through subtracting the objective function value from unit.

5. RESULTS AND DISCUSSION

There are various types of biomass resources which can be used to produce biofuel such as sunflower, sorghum, Jatropha, sugar beet, and Algae. It should be noted that the ecological condition and refinery facilities in Iran are suitable for biofuel production from alga. Also, availability of high seas in Iran facilitates the possibility of cultivation of algae.

In this study, we use economic, social, and environmental factors which are triple lines of sustainable development for investigating the appropriateness of different provinces in Iran for algae cultivation. We measured nine effective indicators including six desirable outputs, *annual rainfall* ($\tilde{g}_1$), *solar radiation* ($\tilde{g}_2$), *amount of water resources* ($\tilde{g}_3$), *annual average of mean daily temperature* ($\tilde{g}_4$), *amount of wastewater* ($\tilde{g}_5$) and *Population* ($\tilde{g}_6$) which should be maximized along with three undesirable outputs criteria including *cultivation cost per hectare* ($\tilde{b}_1$), *distance to refineries* ($\tilde{b}_2$) and *human development index* ($\tilde{b}_3$), which have been produced but should be minimized. The used indicators have been presented and discussed by Babazadeh et al. (2020). There are 30 DMUs including different provinces of Iran for microalgae cultivation. The northern and southern provinces of Iran have excellent conditions such as more water, good humid and high temperature to be utilized for microalgae cultivation. Meanwhile, other provinces of Iran have good conditions for algae cultivation. The aim of this paper is to determine the best locations according to economic, environmental and social factors. As now, successful cultivation of microalga has been done in some provinces of Iran such as Golestan, Gilan, Bushehr, and Hormozgan.

5.1 The Results of the FDEA Model

We apply the FDEA model on the data driven from a real case study in Iran. Table 2 summarizes the UNEs and ranks obtained for various amounts of α-cut levels. GAMS (2018) optimization software is employed to solve the FDEA model.

As can be adapted from the table, Hormozgan (DMU28) achieved the best ranking score for $\alpha \in \{0.4, 0.7, 0.9, 1\}$ while it is ranked as the 4th of the ranking for $\alpha \in \{0, 0.1\}$. As a matter of fact, Kohgiluyeh and Boyer-Ahmad (DMU22) is the 1st, 2nd, and 3rd of the ranking for $\alpha \in \{0, 0.1\}$, $\alpha \in \{0.4\}$, and $\alpha \in \{0.7, 0.9, 1\}$, respectively. Moreover, Khorasan Sh. (DMU11) is ranked as the 30th position for all the employed α, i.e $\alpha \in \{0, 0.1, 0.4, 0.7, 0.9, 1\}$, which shows an identical ranking score for various α. In contrast, the maximum variation in ranking scores for different α is associated to Khorasan R. (DMU10) and Yazd (DMU30). The former province is ranked as the 8th position for $\alpha = 1$ while it possesses 25th ranking score with $\alpha \in \{0, 0.1, 0.4\}$. The latter one is recognized as the 14th and 21st positions for $\alpha = 1$ and $\alpha = 0$, respectively. The achieved results are consistent with real experiences so that according to the studies of Iran Fisheries Organization, the Hormozgan has been recognized the most suitable location for microalgae cultivation. At the other hand, since the water availability is crucial for successful cultivation of microalgae, the FDEA model has eliminated the provinces dealing with water shortage.

Table 2. The results of the model for various amounts of α

No	DMUs (Provinces)	$\alpha = 0$ UNE	Rank	$\alpha = 0.1$ UNE	Rank	$\alpha = 0.4$ UNE	Rank	$\alpha = 0.7$ UNE	Rank	$\alpha = 0.9$ UNE	Rank	$\alpha = 1$ UNE	Rank
1	Azerbaijan Sh.	0.61	20	0.61	21	0.63	22	0.65	23	0.67	23	0.68	23
2	Azerbaijan Gh.	0.62	17	0.62	19	0.64	20	0.66	20	0.68	21	0.68	22
3	Ardabil	0.61	19	0.62	18	0.64	19	0.67	18	0.68	20	0.69	21
4	Isfahan	0.63	16	0.64	16	0.65	17	0.69	17	0.74	16	0.76	15
5	Ilam	0.65	14	0.66	14	0.72	13	0.80	10	0.85	10	0.88	9
6	Bushehr	0.74	11	0.75	10	0.80	9	0.84	9	0.88	8	0.89	8
7	Tehran	0.84	5	0.84	5	0.86	6	0.88	6	0.89	7	0.90	7
8	Chahar Mahaal and Bakhtiari	0.74	10	0.75	9	0.77	10	0.79	12	0.81	12	0.82	12
9	Khorasan J.	0.52	27	0.53	27	0.56	27	0.60	26	0.64	26	0.66	25
10	Khorasan R.	0.59	25	0.60	25	0.62	25	0.65	22	0.68	19	0.70	18
11	Khorasan Sh.	0.50	30	0.51	30	0.53	30	0.55	30	0.56	30	0.57	30
12	Khozestan	0.76	8	0.77	8	0.80	8	0.84	8	0.86	9	0.87	10
13	Zanjan	0.53	26	0.54	26	0.56	26	0.58	27	0.59	28	0.60	28
14	Semnan	0.50	29	0.51	29	0.53	29	0.56	28	0.59	27	0.60	27
15	Sistan va Balochestan	0.78	7	0.80	7	0.83	7	0.87	7	0.89	6	0.91	6
16	Fars	0.80	6	0.82	6	0.87	5	0.93	2	0.96	2	0.98	2
17	Gazvin	0.50	28	0.51	28	0.53	28	0.55	29	0.57	29	0.57	29
18	Gom	0.60	22	0.60	24	0.63	21	0.65	21	0.67	22	0.69	20
19	kordestan	0.64	15	0.65	15	0.67	15	0.70	16	0.71	17	0.72	17
20	Kerman	0.66	13	0.67	13	0.73	12	0.79	11	0.82	11	0.85	11
21	Kermanshah	0.74	9	0.74	11	0.76	11	0.78	13	0.79	13	0.79	13
22	Kohgiluyeh and Boyer-Ahmad	0.87	1	0.88	1	0.90	2	0.92	3	0.93	3	0.94	3
23	Golestan	0.61	18	0.62	17	0.64	18	0.66	19	0.68	18	0.69	19
24	Gilan	0.86	3	0.87	3	0.89	4	0.91	4	0.92	4	0.93	4
25	Lorestan	0.68	12	0.69	12	0.71	14	0.73	14	0.75	14	0.75	16
26	Mazandaran	0.86	2	0.87	2	0.89	3	0.90	5	0.91	5	0.92	5
27	Markazi	0.59	24	0.60	23	0.62	24	0.63	25	0.64	25	0.65	26
28	Hormozgan	0.85	4	0.86	4	0.90	1	0.93	1	0.96	1	0.98	1
29	Hamadan	0.59	23	0.60	22	0.62	23	0.64	24	0.66	24	0.67	24
30	Yazd	0.60	21	0.61	20	0.66	16	0.71	15	0.74	15	0.76	14

To apply the FDEA model in real-world practices, it should firstly be verified and validated. The spearman's rank correlation method is an efficient method for verifying and validation of the DEA models (Sheskin, 2000). The ranks achieved by the FDEA and DEA models are assessed in the terms of positive correlation through calculating the measure $\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$. Here, d_i is difference between the ranks achieved from each model and n indicates the total number of DMUs. Table 3 shows the Spearman's rank correlation coefficient for different values of α-cuts. A significant correlation could be observed between the ranks provided by the FDEA and DEA models.

Table 3. Positive correlation between FDEA and DEA models

	α = 0.1	**α = 0.4**	**α = 0.7**	**α = 0.9**
Correlation	0.9484	0.9646	0.9851	0.9931

6. CONCLUSION

Energy plays a fundamental role in human life. Considering the population growth, more food and energy will be needed to ensure an adequate standard of living for all people around the world. Energy demand has been significantly growing every year while the energy resources remain constant. The carbon dioxide emitted due to fossil fuels combustion is increasing rapidly which causes irreversible changes to earth like global warming, air pollution, climate change, and acid rain and its effects on the agricultural industry and human health. Due to the mentioned reasons, today countries are more willing to use renewable energies including biodiesel. Biodiesel generation from non-food raw materials such as algae has become very important in recent years. Determining the optimal areas for algae cultivation reduces the production costs of algae oil and leads to greater profitability of biodiesel supply chains. This paper investigates the growth indicators of algae cultivation to evaluate the efficiency of the candidate areas under epistemic condition. To do so, we extended an FDEA model with the aim of measuring the efficiency scores of locations under uncertainty. The places with high rank are appropriate ones in order to calculate algae. A real case study is conducted in Iran to verify and validate the results of the FDEA and DEA models. The obtained results pointed out that the suggested FDEA model can be assistance to the decision makers for optimizing the cultivation locations of algae for various levels of acceptable uncertainty (α-cut). For the future researches, one can extend our approach to compare its results with other approaches such as stochastic or robust DEA models.

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CONFLICT OF INTEREST STATEMENT
On behalf of all authors, the corresponding author states that there is no conflict of interest.

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